Particle Physics – Neutrino Mass Generation via the See-Saw Mechanism

Thurston Matthews

September 2024

Abstract

The see-saw mechanism is a compelling theoretical framework addressing the small but non-zero masses of neutrinos by introducing heavy right-handed neutrinos. This mechanism elegantly explains the lightness of neutrino masses compared to other fermions and has profound implications for cosmology and particle physics.

---

1 Mass Matrix Construction

1.1 Neutrino Mass Terms: Dirac and Majorana

In the Standard Model (SM), neutrinos are massless because only left-handed neutrinos and right-handed antineutrinos are present, and there are no right-handed neutrino fields [1]. To generate neutrino masses, we extend the SM by introducing right-handed neutrino fields N_R. This allows for both Dirac and Majorana mass terms.

1.1.1 Mass Terms in the Lagrangian

The most general mass terms involving neutrinos can be written as:

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{N_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$

where:

- ν_L are the left-handed neutrino fields.
- N_R are the introduced right-handed neutrino fields.
- m_D is the Dirac mass matrix, arising from the Yukawa coupling after electroweak symmetry breaking:

  $$m_D = Y_\nu \langle H \rangle$$

  where Y_ν are the Yukawa couplings and $\langle H \rangle$ is the Higgs vacuum expectation value (VEV).
- M_R is the Majorana mass matrix for the right-handed neutrinos, which is not forbidden by any SM gauge symmetry and can naturally be much larger than the electroweak scale [2].

1.1.2 Origin of Each Term

- **Dirac Mass Term (m_D)**: Couples left-handed neutrinos ν_L with right-handed neutrinos N_R, analogous to mass terms for other fermions in the SM. It originates from the Yukawa interactions after the Higgs field acquires a vacuum expectation value (VEV).
- **Majorana Mass Term (M_R)**: Involves only the right-handed neutrinos and does not require the Higgs mechanism. Since N_R are SM singlets, M_R can be at a very high scale, potentially related to grand unified theories (GUTs) [3,4].

1.1.3 Neutrino Mass Matrix

The combined mass terms form the following neutrino mass matrix $\mathcal{M}$:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

This matrix is a **Majorana mass matrix** because it includes both Dirac and Majorana mass terms, allowing for the possibility of Majorana neutrinos (particles that are their own antiparticles).

---

2 Diagonalization

2.1 Objective

To find the mass eigenstates and their corresponding masses, we need to diagonalize the mass matrix $\mathcal{M}$.

2.2 See-Saw Mechanism Approximation

Assuming that the Majorana mass scale M_R is much larger than the Dirac mass scale m_D ($M_R \gg m_D$), we can apply the see-saw approximation [5].

2.2.1 Block Matrix Diagonalization

Given:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

Under the assumption $M_R \gg m_D$, the mass matrix can be approximately diagonalized as follows:

1. **Leading Order Eigenvalues**
   - **Heavy Eigenstates**: Approximately given by M_R. These correspond to predominantly right-handed neutrinos with masses near the scale M_R.
   - **Light Eigenstates**: Generated by an effective mass matrix for the left-handed neutrinos:

     $$m_\nu \approx -m_D M_R^{-1} m_D^T$$

2. **Eigenstates**
   - **Heavy Neutrinos (N)**: Mass eigenstates with masses $\sim M_R$.
   - **Light Neutrinos (ν)**: Mass eigenstates with masses $\sim m_\nu$, which are much smaller than M_R.

2.3 Inversion of Heavy Majorana Masses

The effective mass of the light neutrinos is inversely proportional to the heavy Majorana masses. This is the essence of the see-saw mechanism:

$$m_\nu \propto \frac{m_D^2}{M_R}$$

Given that M_R is very large (e.g., 10^{14} GeV) and m_D is of the order of typical fermion masses (e.g., electroweak scale), this naturally explains why neutrino masses are so small.

2.4 Diagonalization Process

For a more detailed diagonalization, consider $\mathcal{M}$ in the basis where M_R is diagonal. The steps involve:

1. **Assume M_R is diagonal**: $M_R = \text{diag}(M_1, M_2, \ldots, M_n)$.
2. **Block Diagonalize $\mathcal{M}$**:

   Using a unitary transformation U to rotate $\mathcal{M}$ into a diagonal form, leading to light and heavy mass eigenstates.
3. **Obtain Light Neutrino Mass Matrix**:

   $$m_\nu = -m_D M_R^{-1} m_D^T$$

   This effective mass matrix m_ν is typically much smaller than the Dirac masses m_D, consistent with observed neutrino masses [1].

---

3 Leptogenesis Implications

3.1 Baryon Asymmetry of the Universe (BAU)

The universe exhibits a matter-antimatter asymmetry, quantified by the baryon asymmetry η_B. One compelling explanation involves the generation of a lepton asymmetry, which is then partially converted into a baryon asymmetry via sphaleron processes—a mechanism known as **leptogenesis** [6].

3.2 Mechanism of Leptogenesis via Heavy Right-Handed Neutrinos

1. **Decay of Heavy Neutrinos**:

   The heavy right-handed neutrinos N_R can decay into Standard Model particles, such as:

   $$N_R \to \ell + H$$

   $$N_R \to \bar{\ell} + H^\dagger$$

   where ℓ represents lepton doublets and H the Higgs doublet.
2. **CP Violation**:

   If these decays violate CP symmetry (i.e., rates for $N_R \to \ell H$ and $N_R \to \bar{\ell} H^\dagger$ are different), a net lepton number can be generated.
3. **Out-of-Equilibrium Decays**:

   For leptogenesis to be effective, the decays must occur out of thermal equilibrium, satisfying one of Sakharov's conditions.
4. **Sphaleron Processes**:

   Electroweak sphaleron transitions violate $B + L$ but conserve $B - L$. These processes can convert the generated lepton asymmetry L into a baryon asymmetry B.

3.3 Quantitative Aspect

The generated baryon asymmetry is proportional to the CP-violating parameter and inversely proportional to the masses of the heavy neutrinos. The see-saw mechanism's natural suppression via large M_R can be compensated by the CP-violating phases in the neutrino sector, allowing for successful leptogenesis [7].

3.4 Connection to See-Saw Mechanism

The same heavy right-handed neutrinos responsible for generating light neutrino masses via the see-saw mechanism are integral to leptogenesis. Their out-of-equilibrium decays and CP-violating interactions provide a natural source for the matter-antimatter asymmetry observed today.

---

4 Experimental Signatures

Detecting heavy right-handed neutrinos directly is challenging due to their high mass scales, often far beyond the reach of current colliders. However, several indirect and potentially direct signatures could provide evidence for their existence and validate the see-saw mechanism:

4.1 Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

- **Description**: A process where a nucleus decays by emitting two electrons without accompanying neutrinos, indicating lepton number violation.
- **Connection to See-Saw**: The observation of $0\nu\beta\beta$ would confirm that neutrinos are Majorana particles, as predicted by the see-saw mechanism [8].
- **Experimental Status**: Ongoing experiments like GERDA, KamLAND-Zen, and EXO-200 are searching for this decay with increasing sensitivity.

4.2 Precision Measurements of Neutrino Masses and Mixings

- **Description**: Precise determination of the neutrino mass hierarchy, absolute mass scale, and mixing angles.
- **Connection to See-Saw**: Any deviations from the Standard Model predictions could hint at the underlying mechanism generating neutrino masses, potentially supporting the see-saw framework.
- **Experimental Status**: Experiments such as KATRIN aim to measure the absolute neutrino mass [9], while neutrino oscillation experiments (e.g., NOvA, DUNE) probe mixing parameters.

4.3 Lepton Flavor Violation (LFV) Processes

- **Description**: Processes like $\mu \to e\gamma$, $\mu \to 3e$, and $\mu - e$ conversion in nuclei.
- **Connection to See-Saw**: Heavy right-handed neutrinos can induce LFV through loop diagrams. Observing LFV at rates higher than SM predictions could signal new physics related to the see-saw mechanism [10].
- **Experimental Status**: Experiments like MEG [11], Mu3e, and COMET are actively searching for LFV processes.

4.4 Direct Production at Colliders

- **Description**: Production of heavy right-handed neutrinos via processes like $pp \to W^* \to \ell N_R$, followed by N_R decaying into SM particles.
- **Connection to See-Saw**: Detection of heavy neutrinos would be direct evidence for the see-saw mechanism. However, the required energies to produce very heavy N_R (e.g., 10^{14} GeV) are beyond current colliders.
- **Possible Scenarios**: If right-handed neutrinos are lighter (e.g., TeV scale), perhaps in extended models like the inverse see-saw, they might be accessible at the LHC or future colliders [12].

4.5 Cosmological and Astrophysical Observations

- **Description**: Impact of heavy neutrinos on the cosmic microwave background (CMB), Big Bang nucleosynthesis (BBN), and structure formation.
- **Connection to See-Saw**: Heavy neutrinos influence the thermal history of the universe and could leave imprints on cosmological observables.
- **Experimental Status**: Precision cosmological data from missions like Planck constrain neutrino properties [13].

4.6 Indirect Effects in Neutrino Oscillations

- **Description**: Subtle deviations from standard neutrino oscillation patterns due to the presence of heavy neutrino states.
- **Connection to See-Saw**: Mixing between light and heavy neutrinos could modify oscillation probabilities or lead to new oscillation phenomena [14].
- **Experimental Status**: Current and future neutrino experiments could probe such effects with high precision.

Appendix: Glossary

- **Baryon Asymmetry of the Universe (BAU)**: The observed imbalance between matter and antimatter in the universe, where matter significantly outweighs antimatter.
- **Big Bang Nucleosynthesis (BBN)**: The production of light atomic nuclei during the early phases of the universe shortly after the Big Bang.
- **Charge-Parity (CP) Violation**: A phenomenon where the laws of physics change when particles are replaced with their antiparticles (charge conjugation) and their spatial coordinates are inverted (parity transformation).
- **Cosmic Microwave Background (CMB)**: The thermal radiation left over from the time of recombination in Big Bang cosmology, acting as a snapshot of the early universe.
- **Dirac Mass**: A type of mass term in the Lagrangian that couples left-handed and right-handed fermion fields, preserving lepton number.
- **Electroweak Symmetry Breaking**: The process by which the Higgs field acquires a vacuum expectation value, giving mass to W and Z bosons and fermions.
- **Fermions**: Particles that follow Fermi-Dirac statistics, including quarks and leptons, which make up matter.
- **Grand Unified Theories (GUTs)**: Theoretical frameworks that attempt to unify the electromagnetic, weak, and strong nuclear forces into a single force.
- **Higgs Vacuum Expectation Value (VEV)**: The non-zero value of the Higgs field in its lowest energy state, responsible for giving mass to particles.
- **Inverse See-Saw Mechanism**: An extension of the see-saw mechanism that introduces additional neutrino states to achieve light neutrino masses with lower-scale right-handed neutrinos.
- **Lepton Flavor Violation (LFV)**: Processes in which a lepton changes its flavor (type), violating the conservation of lepton flavor numbers.

- **Leptogenesis**: A theoretical process that explains the matter-antimatter asymmetry of the universe by generating a lepton asymmetry that is partially converted into a baryon asymmetry.
- **Majorana Mass**: A mass term that couples a fermion field to its own charge-conjugate, allowing particles to be their own antiparticles.
- **Majorana Neutrinos**: Neutrinos that are their own antiparticles, described by Majorana mass terms.
- **Mass Eigenstates**: The states of particles with definite mass, obtained after diagonalizing the mass matrix.
- **Neutrino Oscillations**: The phenomenon where neutrinos change their flavor as they propagate, due to differences in their mass eigenstates.
- **Neutrinos**: Elementary particles with very small mass and no electric charge, coming in three flavors corresponding to the electron, muon, and tau leptons.
- **Neutrinoless Double Beta Decay ($0\nu\beta\beta$)**: A hypothetical nuclear process that, if observed, would indicate that neutrinos are Majorana particles and that lepton number is not conserved.
- **Right-Handed Neutrinos (N_R)**: Hypothetical neutrino states that are singlets under the Standard Model gauge group, introduced to explain neutrino masses.
- **Sakharov Conditions**: Three necessary conditions outlined by Andrei Sakharov for generating a matter-antimatter asymmetry: baryon number violation, C and CP violation, and departure from thermal equilibrium.
- **See-Saw Mechanism**: A theoretical model explaining the smallness of neutrino masses by introducing heavy right-handed neutrinos, leading to an inverse relationship between light and heavy neutrino masses.
- **Sphaleron Processes**: Non-perturbative electroweak processes that violate baryon and lepton numbers but conserve $B - L$, active at high temperatures in the early universe.
- **Standard Model (SM)**: The theory describing the electromagnetic, weak, and strong nuclear interactions, accounting for all known elementary particles.
- **Yukawa Coupling**: Interaction terms in the Lagrangian between fermions and scalar fields (like the Higgs), responsible for generating fermion masses after symmetry breaking.

Conclusion

The see-saw mechanism offers a robust framework for understanding the smallness of neutrino masses by introducing heavy right-handed neutrinos. Its implications extend beyond mass generation, providing potential explanations for the baryon asymmetry of the universe through leptogenesis and predicting various experimental signatures that could validate this elegant theoretical construct. Ongoing and future experiments across particle physics and cosmology are crucial in testing the predictions of the see-saw mechanism and uncovering the fundamental nature of neutrinos.

Acknowledgments

The author acknowledges the assistance of ChatGPT in the preparation of this manuscript.

References

[1] C. Giunti and C. W. Kim, *Fundamentals of Neutrino Physics and Astrophysics*, Oxford University Press, 2007.

[2] R. N. Mohapatra and G. Senjanović, *Neutrino Mass and Spontaneous Parity Nonconservation*, Phys. Rev. Lett. **44**, 912 (1980).

[3] M. Gell-Mann, P. Ramond, and R. Slansky, *Complex Spinors and Unified Theories*, In *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman, North Holland, Amsterdam, 1979.

[4] T. Yanagida, *Horizontal Gauge Symmetry and Masses of Neutrinos*, Prog. Theor. Phys. **64**, 1103 (1980).

[5] P. Minkowski, *→ e at a Rate of One Out of 1-Billion Muon Decays?*, Phys. Lett. B **67**, 421 (1977).

[6] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, Phys. Lett. B **174**, 45 (1986).

[7] W. Buchmüller, R. D. Peccei, and T. Yanagida, *Leptogenesis as the Origin of Matter*, Annu. Rev. Nucl. Part. Sci. **55**, 311 (2005).

[8] J. D. Vergados, H. Ejiri, and F. Šimkovic, *Neutrinoless Double Beta Decay and Neutrino Masses*, Rep. Prog. Phys. **75**, 106301 (2012).

[9] KATRIN Collaboration, *First Tritium Data from the Karlsruhe Tritium Neutrino Experiment*, Phys. Rev. Lett. **123**, 221802 (2019).

[10] M. Raidal *et al.*, *Flavour Physics of Leptons and Dipole Moments*, Eur. Phys. J. C **57**, 13 (2008).

[11] MEG Collaboration, *New Constraint on the Existence of the + → e+ Decay*, Phys. Rev. Lett. **110**, 201801 (2013).

[12] A. Atre, T. Han, S. Pascoli, and B. Zhang, *The Search for Heavy Majorana Neutrinos*, J. High Energy Phys. **05**, 030 (2009).

[13] Planck Collaboration, *Planck 2018 Results. VI. Cosmological Parameters*, Astron. Astrophys. **641**, A6 (2020).

[14] A. Abada, C. Biggio, F. Bonnet, M. Gavela, and T. Hambye, *Low Energy Effects of Lepton Number Violation*, J. High Energy Phys. **12**, 061 (2007).